Beyond the Partial-Adjustment Model

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Firms and consumers do not respond frequently to changes in their environment, and when they respond, adjustments are typically large.[1] It is also reasonable to postulate that the probability with which these adjustments take place depends on the departure of the economic agent's main state variable from what its level would be in the absence of adjustment costs.

Deriving the macroeconomic consequences of the *state-dependent* microeconomic behavior described above is generally difficult. It involves keeping track of the fraction of units adjusting in any given period, a fraction whose evolution is determined endogenously over time (see e.g., Alan S. Blinder, 1981; Andrew S. Caplin and Daniel Spulber, 1987; Giuseppe Bertola and Caballero, 1990; Caballero and Engel, 1991; Caplin and John Leahy, 1991). One exception, in which the aggregation problem is trivial, is the constant-adjustment *hazard* case (i.e., the case in which the probability of adjustment is constant and therefore unrelated to the distance of microeconomic state variables from their targets). This case yields the celebrated *partial-adjustment model*, which can also be obtained from a model in which a representative agent faces quadratic adjustment costs.[2]

Unfortunately, there are many reasons to doubt that microeconomic adjustment hazards are constant. For instance, unless one maintains that economic agents tolerate small and large departures alike, it is likely that at some point the probability of adjusting begins to increase with the distance between the state variables and the underlying frictionless optima. In this paper we go beyond appealing to micro foundations, however, and argue that abandoning the simplicity of the partial-adjustment model in favor of *increasing hazard models* is empirically relevant. We show that increasing-hazard models account for richer aggregate dynamics, especially during large recessions and brisk expansions. We also show how these models provide a natural and simple framework in which microeconomic data can be used to enrich the description of aggregate dynamics.

After sketching the basic features and equations of aggregate state-dependent models based on hazard functions, we illustrate the relevance of these models with an application to U.S. manufacturing employment data. We estimate the corresponding hazard function and find that it is asymmetric and eventually increases with the distance from the optimal target. More importantly, the hazard model we estimate not only outperforms the constant-hazard model, but it does so by improving the explanatory power during large shock episodes, which is precisely when one would expect nonlinearities to matter most.[3]

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[1]See, for example, Daniel S. Hamermesh (1989) and Steven J. Davis and John Haltiwanger (1990) for evidence on lumpy changes in plant-level employment.

[2]See Guillermo A. Calvo (1983) for a version of the constant-hazard model, and see Julio J. Rotemberg (1987) for a proof of the aggregate equivalence of this model and the quadratic/representative-agent model. One can also show that the dynamics of the constant-hazard model are equivalent to those of *time*-dependent models with uniform staggering.

[3]Caballero (1990b) provides an example showing how the probabilistic mechanisms underlying state-dependent models tend to offset the aggregate effects of nonlinearities at the microeconomic level. It follows

I. Basic Equations

In this section, we briefly describe the basic equations underlying the preceding discussion. Since our empirical example is based on employment data, we describe these equations in terms of firms' employment decisions; of course the framework is more general than this particular application. For example, it can be used to study the dynamic behavior of business and residential investment, consumer-durables expenditures, inventory investment, and the price level.

We let e_{it} and e_{it}^* denote actual and frictionless employment (all variables in logarithms unless otherwise stated) within firm $i \in [0,1]$ at time t, and define z_{it} as the difference between these two quantities: $z_{it} \equiv e_{it} - e_{it}^*$. In the absence of frictions we have $z_{it} = 0$ for all t. We start from the presumption that the frictionless scenario is unrealistic and assume that e_{it} is not controlled continuously. The most obvious justification for this assumption is a nonconvexity, such as a fixed cost, in the adjustment-cost function. The best known example of this type of models is the (S,s) family. For example, in the particular case of the (L,C,U) model, the microeconomic unit does not act until the departure of its state variable from its optimal value, a difference we denote by z and call the unit's deviation, reaches its lower trigger L or its upper trigger U. At these points, the unit adjusts, thereby bringing z to the target level C.

Instead of using rigid (L,C,U) policies, we describe firms' behavior in terms of a more general probabilistic rule. We assume that the probability that firm i adjusts its level of employment during the time interval $(t, t+dt]$ is equal to $\Lambda(z_{it})\,dt$, where $\Lambda(z)$ denotes the *adjustment-hazard function*. All we require of $\Lambda(z)$ is that it eventually increases with the distance of z to the target level C. This is much weaker than the extreme case of an (L,C,U) policy, where the hazard function jumps from zero to infinity at the trigger levels.

There are many microeconomic justifications for considering increasing hazard functions instead of the particular case of (L,C,U)-type policies. These range from time-varying fixed adjustment costs and rich properties of the e_{it}^* process to near-rationality-type arguments.[4] Furthermore, considering this general family of models is particularly useful for empirical purposes, since it includes a wide variety of models, among them the partial-adjustment and (S,s) models.

Once the hazard function is nonconstant, it becomes important to keep track of the evolution of the cross-section distribution of firms' z_i's over time. For example, the response of aggregate labor demand to an aggregate shock will depend on whether most units are within regions with large or small adjustment hazards.

For simplicity we work in discrete time. Dropping a constant that is irrelevant for our dynamic analysis, we assume that the target z is zero, and we assume that the firm's employment level changes by $-z_{it}$ when it adjusts. Thus, the change in aggregate labor demand in $(t, t+1]$ is equal to:

$$(1) \quad \Delta E_{t+1} = -\int_{-\infty}^{\infty} (z - \Delta A_{t+1})\Lambda(z - \Delta A_{t+1}) f(z,t)\,dz$$

where $\Delta X_{t+1} \equiv X_{t+1} - X_t$, and E_t and $f(z,t)$ denote aggregate employment and the cross-section distributions of the z_i's at time t, respectively.

To derive (1), we first consider the fraction $f(z,t)$ of firms that would adjust by z at time t in the absence of adjustment costs. After an aggregate shock ΔA_{t+1} takes place,

that these nonlinearities are likely to permeate the aggregate dynamics only when shocks are large.

[4] Hazard functions that are not increasing are also useful. For example, when hazard functions are increasing but different across firms, then the evolution of aggregate labor demand can be approximated by a hazard model with hazard function equal to a weighted average of individual hazards. This average is not necessarily increasing.

these firms' z_i's change to $z - \Delta A_{t+1}$; a fraction $\Lambda(z - \Delta A_{t+1})$ among them adjust after the "hazard shock" takes place, all of them by $-(z - \Delta A_{t+1})$. Adding over all possible values of z then leads to (1). Idiosyncratic shocks other than those coming from the realization of the hazard shock take place next, affecting the cross-section distribution in place for the next sequence of shocks.[5]

Simple algebraic steps take us from equation (1) to what we believe is a very useful expression for applied work:

$$(2) \qquad \Delta E_{t+1} = \sum_{k \geq 0} a_k(\Delta A_{t+1}) Z_t^{(k)}$$

where the $a_k(\cdot)$ functions can be obtained explicitly (see Caballero and Engel [1992] and the examples below), and $Z_t^{(k)}$ denotes the kth moment of the cross-section distribution of z_i's at time t.

In the constant-hazard case $[\Lambda(z) \equiv \lambda_0]$, only $a_0(\cdot)$ and $a_1(\cdot)$ are nonzero, implying

$$(3) \qquad \Delta E_{t+1} = \lambda_0\big(\Delta A_{t+1} - Z_t^{(1)}\big) = \lambda_0(A_{t+1} - E_t)$$

and thus obtaining the standard partial-adjustment equation.[6] The quadratic hazard case, $\Lambda(z) = \lambda_0 + \lambda_2 z^2$, on the other hand, is an example of an increasing-hazard model. Here,

$$\Delta E_{t+1} = \Delta E_{t+1}^q - \lambda_2 \times \big\{-\Delta A_{t+1}^3 + 3\Delta A_{t+1}^2 Z_t^{(1)} - 3\Delta A_{t+1} Z_t^{(2)} + Z_t^{(3)}\big\}$$

where ΔE^q represents the expression for ΔE in the partial-adjustment model. This equation shows that higher moments of the cross-section distribution matter when the hazard function is nonconstant. Considering the Taylor expansion of more general hazard functions, we can see how higher moments of the cross-section distribution play a role when determining the dynamic behavior of aggregate employment.[7]

II. Application

In this section, we estimate a hazard function for U.S. manufacturing employment changes for the period 1961:1–1983:1.[8] We stress that this is intended to be an illustrative example, not a thorough explanation of U.S. labor markets.

The first step is to construct a series of aggregate shocks, ΔA_t. We describe the procedure for this in detail in Caballero and Engel (1992). The basic idea is that, because employment does not fully adjust to shocks, the observed changes in output are on average smaller than the actual aggregate shocks. We proxy the extent of this smoothing by the changes in average hours worked, which we assume would be constant in a world without frictions. The resulting equation is $\Delta A_t = \Delta Y_t + \theta \Delta H_t$, where Y denotes manufacturing output and H denotes average hours worked by manufacturing production workers. We set $\theta = 5$, which corresponds to a markup coefficient of 12 percent and a short-run marginal cost elasticity of 0.3. Reasonable changes in θ do not affect our conclusions.

We first estimate the partial-adjustment/constant-hazard model in equation (3). We obtain a quarterly probability of adjusting equal to 0.14 ($\lambda_0 = 0.14$) and an R^2 of 0.66. Next we estimate an increasing-hazard model. From the discussion above, it follows that to do this we either need additional data or we must simulate the higher moments of the cross-section distribution. We begin with the latter approach. We postulate that aggregate and firm-specific shocks

[5]This timing convention is innocuous.

[6]Two steps of simple algebra transform this equation into $\Delta E_{t+1} = \lambda_0 \Delta A_{t+1} + (1 - \lambda_0)\Delta E_t$, that is, another representation of the partial-adjustment model.

[7]More efficient methods of approximation than high-order polynomials (Taylor expansions) can be used when one has additional information or strong priors on the actual shape of the hazard function.

[8]This is the maximum period for which we had all the data needed for the different procedures we implement in this paper. All the data come from CITIBASE.

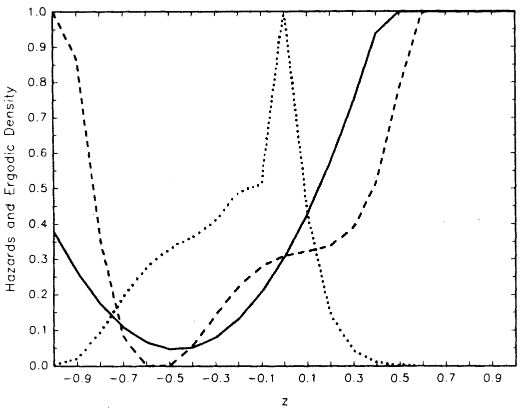

FIGURE 1. ESTIMATED HAZARD FUNCTIONS

Notes: The solid line is the aggregate hazard function estimated with mathematical simulation of moments; the dashed line depicts the estimated hazard function using moments of the cross-section distribution of two-digit SIC industries. The dotted line depicts the corresponding "average" cross-section distribution.

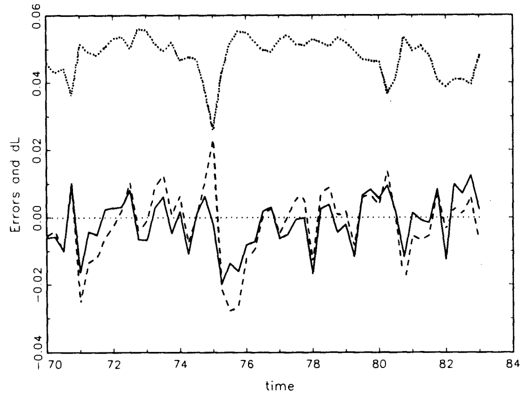

FIGURE 2. ERRORS FROM THE HAZARDS MODEL

Notes: The solid line depicts the negative of the error from the nonconstant-hazard model; the dashed line depicts the negative of the error from the model with partial adjustment using two-digit SIC industry data. The dotted line at the top of the graph shows a transformation of actual changes in employment $(0.04+0.3\ dL)$.

are driven by random walks and use the associated Kolmogorov equation to track the simulated cross-sectional distribution and its moments.[9] The results are encouraging. Higher moments clearly matter and the R^2 rises to 0.84. The solid line in Figure 1 depicts the estimated hazard function, which is clearly nonconstant and belongs to the increasing-hazard family.[10]

Next we estimate the hazard function using additional data instead of mathematical simulation of moments. Unfortunately we do not have individual firms' z_i's. We construct estimates of the path of the required moments from the moments of the cross-section distribution of two-digit SIC industries. Since this is likely to smooth the variability of the moments, we add a free proportionality factor, v, to the hazard function, $\Lambda(vz)$. We estimate this constant by minimizing the distance from the hazard estimated with the mathematical simulation procedure.[11] We approximate a general hazard function by the first five terms of its Taylor expansion and construct the corresponding equation (2). The results are again encouraging: Higher moments are highly significant, the hazard function is clearly nonconstant, and the R^2 is 0.82. The dashed line in Figure 1 illustrates the shape of the estimated hazard function, which has the same general features of the hazard estimated with mathematical simulation of moments. The dotted line depicts the corresponding "average" cross-section distribution, which shows that the nonconstant segments of the hazard occur in relevant regions.

It is important to point out that the mere existence of a nonconstant hazard is in general not sufficient to gain explanatory power over the partial-adjustment model. The real gain comes when the nonconstant hazard is combined with large shocks. Figure 2 plots the negative of the errors obtained from the constant and flexible (mathematical simulation case) hazards models estimated above. It also plots the path of actual changes in

[9]See Caballero (1990a) and Caballero and Engel (1992) for examples of this type of procedure.

[10]The estimated hazard is of the form: $\Lambda(z)=\lambda_0+\lambda_1 z+\lambda_2 z^2$. Higher-order terms did not improve the fit significantly.

[11]If we remove the proportionality factor (i.e., set $v=1$), the two-digit hazard becomes (artificially) tighter than the mathematical simulation hazard; however, they still have the same shape.

employment, which is used as a business-cycle clock for the residuals.[12] The most interesting feature of this figure is that the additional explanatory power obtained by using an increasing-hazard model (instead of the partial-adjustment model) appears when large shocks take place, like the recession of 1975 and its recovery. It is precisely during these "large shock episodes" that relevant nonlinear models should be valuable for understanding aggregate dynamics.

III. Conclusion

In a sense, this paper answers the following questions: How well does the partial-adjustment model perform in realistic situations? Can we identify particular circumstances in which the partial-adjustment shortcut performs badly, and should this be the case, is there an alternative model we can use instead?

Our answer to these questions is that, when adjustment hazards are not constant, as is the case for U.S. manufacturing employment, the partial-adjustment model does a good job at tracking the dynamics during normal times. However, it fails during periods of large fluctuations, like deep recessions and brisk expansions.

Increasing-hazard models provide a natural generalization of the partial adjustment model, that performs equally well during normal times and outperforms it during periods with large shocks. These models can be estimated either by simulating moments or by using information from disaggregated data on moments of the cross-section distribution. New microeconomic data sets and our better understanding of the mathematics of cross-section distributions are both such that these amendments to the partial-adjustment model are now easy to implement. The payoff is high, especially when one considers that we are just beginning to see the empirical rewards of adopting models with more realistic microeconomic adjustment behavior.

[12] We report only the second half of our data to provide a clearer figure. Changes in employment are scaled and shifted.

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