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Toll Competition Among Congested Roads

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Eduardo M. Engel, Ronald Fischer, and Alexander Galetovic

Abstract

Roads are being franchised to private firms in many countries, raising the issue of regulating the tolls they charge. When there is more than one road to get from one point to another, regulation need not be necessary, since competition may substitute for toll regulation. This paper studies toll competition among private asymmetric roads subject to congestion. We obtain two main results. First, in equilibrium tolls are higher than optimal, that is, there is too little congestion. This happens because road owners internalize the reduction in drivers' willingness to pay due to congestion, thereby softening competition. It follows that the drawback of private competition is exercise of market power, not excessive congestion as is sometimes conjectured. Second, the distortion becomes smaller as market size and the number of roads grow, even if the density of drivers does not change. In the limit tolls converge to the socially optimal level and are just enough to make each driver internalize the congestion externality. This suggests that the scope for competition is better in larger networks.

KEYWORDS: Private toll roads, congestion, competition

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1 Introduction

In many countries roads are being franchised to private firms.[1] In exchange for toll revenue, the franchise holder finances, builds, operates and maintains the road.[2] When the franchised road has no close substitute, the government must regulate tolls. Yet when there is more than one way to get from one point to another, as is often the case in large cities, competition between several franchise owners might substitute for regulation.[3] This paper studies toll competition among private asymmetric roads subject to congestion. We find that tolls tend to be too high in equilibrium (i.e., there is too little congestion), and that the distortion tends to disappear as market size and the number of roads grows. In fact, in the limit tolls converge to the socially optimal level and are just enough to make each driver internalize the congestion externality.

In our model a pair of locations is joined by several asymmetric roads. Each road is subject to congestion and run by a different operator. Private road owners compete by setting tolls. Drivers choose which road to travel to minimize the generalized travel cost.

We find sufficient conditions for existence of a pure strategy equilibrium with strictly positive tolls. While the franchise holder incurs no direct costs when one additional car uses the road, a lower toll increases the congestion cost borne by users. Since congestion reduces drivers' willingness to pay for using the road, franchise holders partly internalize congestion costs when setting tolls. This softens price competition and the equilibrium is similar to the one that obtains in a standard Cournot game. The cost of congestion acts like the capacity constraint in Kreps and Scheinkman's (1983) two stage oligopoly game, in which capacity is chosen in the first stage and firms compete in prices in the second stage. This runs counter to the intuition that toll competition will lead to Bertrand outcomes and excessive congestion. In fact the opposite is the case and tolls are higher than the socially maximizing tolls. Moreover, we show that the traffic assignment among roads is generally inefficient; that is, taken as given the number of drivers, they

[1]This practice is likely to become more common in the United States as well, because of worsening budgetary problems at the state and local levels.

[2]See Engel, Fischer and Galetovic (2001) for a discussion of highway franchising.

[3]Chile, which has begun to franchise a large fraction of its main roads, exhibits several examples of this situation. A 1400 km stretch of the Panamerican highway (divided into 10 smaller projects) that runs along the country had been franchised by 1998. Several lateral branches to coastal cities were also franchised. In several cases these branches compete with each other. Examples are Nogales-Puchuncaví which competes in the access coastal resorts with the El Melón tunnel on the Panamerican highway and the La Dormida project, which competes with the highway to the major port of Valparaíso. This highway also faces competition from the Ruta del Sol highway from Santiago to the major port of San Antonio, which is close to Valparaíso. The Route to Chicureo, which joins the wealthier sections of Santiago to the Panamerican highway north of Santiago competes with the Route to Los Andes and with the first segment of the Panamerican road north of Santiago. In most of these cases, the roads are not perfect substitutes, but they have made the issue of competition between tolled roads an important topic in the analysis of new franchises in Chile.

could be reallocated and welfare increased. Hence, with a small number of roads the problem is that competition between road owners is too soft–a market power problem–, and not, as is sometimes conjectured, that Bertrand-like competition will lead to excessive congestion.[4]

We also find that distortions tend to disappear when the number of roads increases, even if the density of drivers is kept constant. In the limit tolls are set at the socially optimal level—that is, just enough to make each driver internalize the congestion. And this holds under rather general conditions, including asymmetric roads and franchise holders that price strategically. Hence, even if demand is kept proportional to the network size, the scope for competition is larger in larger networks.

This paper is related to the literature on road pricing pioneered by Walters (1961).[5] Traditionally, this literature has studied road pricing as a standard planning problem.[6] Our model generalizes the one developed by Verhoef, Nijkamp and Rietveld (1996) who studied strategic toll setting by two competing private road owners.[7] It is also related to de Palma and Lindsay (2000), who compare the results of competition among two identical alternative roads under various ownership regimes, and model congestion with Vickrey's (1969) bottleneck model. In this paper we ignore the dynamics of congestion but consider asymmetric roads. Moreover, we study competition when drivers can choose among more than two roads. Last, Viton (1995) has studied whether a private road can profitably compete with an untolled public road.

The paper also suggests a close relation between the economics of franchised roads and the economics of clubs.[8] Roads are subject to congestion, much like standard club goods. As in Scotchmer's (1985a, 1985b) analysis of club goods, this paper models the strategic interaction of road owners and looks for Nash equilibria. Yet roads differ from clubs in two relevant dimensions. First, geographic constraints determine the characteristics of competing roads, thus the symmetry assumption commonly made in the club literature is not justified. Second, the free entry assumption stressed in Scotchmer (1995b) is not realistic for roads either. This paper may therefore be viewed as an extension of the main results in Scotchmer (1985a, 1985b) to the case of a fixed number of competing asymmetric roads. This extension builds on Wardrop's (1952) characterization of equilibrium traffic assignments among roads.

The inefficiency of a price equilibrium has been noted by Levhari and Luski (1978) in the context of waiting times for service by duopolists and by Häckner

[4]See, for example, De Palma (1992).

[5]See Hau (1992) for a survey and Mohring (1994) for a collection of the most important articles.

[6]For early contributions see Lévy-Lambert (1968) and Marchand (1968). Recent contributions are the series of papers by Arnott, de Palma and Lindsey (1990, 1993, 1994).

[7]See also de Palma (1992).

[8]See, for example, Berglas (1976, 1981), Berglas and Pines (1981), Boadway (1980) and Scotchmer (1985a, 1985b).

and Nyberg (1996) in the case of reciprocal externalities, including congestion. We extend these results to the case in which congestion costs are different for the different roads. The inefficiency of traffic assignments under congestion has been noted before (Sheffi, 1985), but it apparently has not been observed to apply to the case of private toll networks. Reitman (1991) has shown that in the case of symmetric congestion costs, road replication in the limit leads to the efficient solution.

The rest of the paper proceeds as follows. Section 2 presents the model. In section 3 we solve the problem of the social planner. Section 4 analyzes competition among roads. Section 5 studies the case where competition increases and finally, section 6 concludes.

2 The model

There are n roads that join two locations. The marginal benefit of an additional trip when Q trips have already been made is $B(Q) \geq 0$, with $B' < 0$, and

$$Q = \sum_{i=1}^{n} q_i, \tag{1}$$

where q_i is the number of trips made on road i. We assume that traffic imposes no maintenance or other costs on the road operator.[9] The cost of making one trip on road i has two components. First, the toll charged by the road operator, p_i; second, the time cost of making a trip when q_i cars are already on the road, $c_i(q_i)$, where $c_i > 0$, $c_i' > 0$, and $c_i'' > 0$. Thus $p_i + c_i(q_i)$ is the generalized travel cost faced by each driver. As is well known since Wardrop (1952), in equilibrium the number of cars on road i is determined by

$$B(Q) = p_i + c_i(q_i); \quad q_i > 0 \tag{2}$$

for all roads i; that is, users will enter roads until the marginal benefit of an additional trip equals the generalized travel cost in each of the n roads. We are ready to examine the social planner's problem.

3 The social planner

In this section we solve the problem of a social planner that can choose both the total number of vehicles travelling, Q, and their distribution on alternative roads,

[9] It is straightforward to extend the present framework to include other costs internalized by each road operator such as road deterioration.

$(q_i)_{i=1}^n$ to maximize

$$\mathcal{S}(q_1, \ldots, q_n) \equiv \int_0^{\sum_{i=1}^n q_i} B(v)dv - \sum_{i=1}^n q_i c_i(q_i). \tag{3}$$

The first term is the sum of benefits that drivers obtain from $Q \equiv \sum_i q_i$ trips. The second term is the sum of the congestion costs borne by drivers. Since tolls redistribute income from users to the owner of the road, they do not affect social surplus directly. Next we show that the objective function (3) is concave and provide conditions for existence and uniqueness of a solution.

Proposition 3.1 *The function $\mathcal{S}(q_1, \ldots, q_n)$ is strictly concave. Furthermore, if for all i*

$$\lim_{q_i \to \infty} c_i(q_i) + q_i c'{}_i(q_i) - B(q_i) > 0, \tag{4}$$

then there exists a unique solution to (3), $q^* \equiv (q_1^*, \ldots, q_n^*)$.[10]

Proof: See the appendix. ∎

Assume that all traffic flows in the planner's solution are positive. It then follows from Proposition 3.1 that the first order sufficient conditions of this problem are

$$\frac{\partial \mathcal{S}}{\partial q_i} = B(\sum_j q_j) - c_i - q_i c_i' = 0; \tag{5}$$

from where for all roads i

$$B(Q^*) = c_i(q_i^*) + q_i^* c_i'(q_i^*), \tag{6}$$

with $Q^* \equiv \sum_i q_i^*$. That is, the benefit derived from the last trip must be equal to the sum of the private cost c_i and the congestion externality $q_i c_i'(q_i)$ in all of the n roads. From the equilibrium condition (2) and the optimality condition (6) it follows that the planner can implement the optimum by charging a set of tolls $(p_i^*)_{i=1}^n$ such that

$$p_i^* = q_i^* c_i'(q_i^*).$$

4 Oligopoly

Consider now the case when each road is owned by a different operator and they compete for traffic by simultaneously choosing tolls. The owner of road i takes

[10] Condition (4) holds, in particular, when $\lim_{q \to \infty} B(q) = 0$.

$(p_j)_{j \neq i}$ as given and chooses p_i to maximize

$$\Pi_i \equiv p_i q_i. \tag{7}$$

In this section we prove existence of a Nash equilibrium in pure strategies and show that in equilibrium the number of vehicles on the road is less than optimal. Moreover, we show with an example that the traffic assignment is inefficient when congestion costs are asymmetric, i.e., the marginal cost of using a road (including congestion costs) is not equalized across roads.

We begin by proving a series of lemmas necessary to prove to the main proposition of the section. Given $p \in \mathsf{R}^n$ we may interpret (2) as defining $q \in \mathsf{R}^n$ as a function of p. We show next that the corresponding inverse function—q as a function of p—is well defined:

Lemma 4.1 *Equation (2) implicitly defines q as a function of p. Furthermore, this function is continuously differentiable in all coordinates.*

Proof: See the appendix. ∎

The next lemma signs the own and cross partial derivatives of traffic with respect to tolls. It also derives identities relating both kinds of derivatives.

Lemma 4.2 *The functions $q_i(p)$, defined implicitly via (2), are such that (i) $\partial q_i/\partial p_i < 0$ and (ii) $\partial q_j/\partial p_i > 0$, for $j \neq i$. Furthermore, we have:*

$$c'_j(q_j)\frac{\partial q_j}{\partial p_i} = 1 + c'_i(q_i)\frac{\partial q_i}{\partial p_i} > 0, \quad j \neq i, \tag{8}$$

$$B'(Q)\sum_{k=1}^{n}\frac{\partial q_k}{\partial p_i} = 1 + c'_i(q_i)\frac{\partial q_i}{\partial p_i} > 0. \tag{9}$$

Proof: See the appendix. ∎

Note that equation (8) implies that $c'_j(q_j)(\partial q_j/\partial p_i)$ does not depend on j, $\forall j \neq i$.

Lemma 4.3 *For all i, $c_i(q) + qc'_i(q)$ is increasing in q.*

Proof: Trivial, given the properties of the cost function. ∎

The following result provides sufficient conditions for the existence of an equilibrium.

Proposition 4.1 *Define $k_i = 1/c_i'(q_i)$ and denote $S_{-i}(k) = \sum_{l \neq i} k_l$. Assume that $B(Q)$ is concave and that*

$$\left[S_{-i}(k) - \frac{1}{B'(Q)}\right]^3 \geq \sum_{j \neq i} \frac{c_j''(q_j)}{c_i''(q_i)} k_j^3, \qquad \forall i, \forall q_i. \tag{10}$$

Then there exists an interior Nash equilibrium in pure strategies.

Proof: See the appendix for the proof and for non trivial examples where the above conditions hold. ∎

We are now ready to prove the main result in this section.

Proposition 4.2 *Let $(p_i^N, q_i^N, Q^N)_{i=1}^n$ be the Nash equilibrium (assumed interior). Then: (i) $Q^N < Q^*$. (ii) The traffic assignment across roads may be inefficient.*

Proof: (i) From the equilibrium condition (2) we have that

$$B(Q^N) - c_i(q_i^N) - q_i^N c'_i(q_i^N) = p_i^N - q_i^N c'_i(q_i^N). \tag{11}$$

Furthermore, an interior Nash equilibrium is a solution to the FOC of the profit equations (7):

$$q_i^N + p_i^N \frac{\partial q_i}{\partial p_i} = 0, \quad \forall i. \tag{12}$$

Using the first order condition (12) to substitute for q_i^N, in equation (11) leads to

$$B(Q^N) - c_i(q_i^N) - q_i^N c'_i(q_i^N) = p_i^N \left[1 + c'_i(q_i^N) \frac{\partial q_i}{\partial p_i}\right] > 0 \tag{13}$$

where the last inequality is due to (9).[11] Now, suppose that, contrary to the proposition, $Q^N \geq Q^*$. Then there exists i such that $q_i^N \geq q_i^*$ and

$$B(Q^N) > c_i(q_i^N) + q_i^N c_i'(q_i^N) \geq c_i(q_i^*) + q_i^* c_i'(q_i^*) = B(Q^*)$$

where the strict inequality follows from (13) and the weak inequality from Lemma 4.3. But since $B'(Q) < 0$, $Q^N < Q^*$, in contradiction with our initial assumption, which completes the first part of the proof.

[11] Some tedious but straightforward algebra shows that this FOC is equivalent to

$$p_i^N = q_i^N \left(c_i' + \frac{1}{-\frac{1}{B'} + \sum_{j \neq i} \frac{1}{c_j'}} \right),$$

which reduces to the well-known expression $p_i^N = q_i^N \left(c_i' + \frac{B' c_j'}{-B' + c_j'} \right)$ for a duopoly.

(ii) To prove the second part of the proposition we present a counterexample that shows that the Nash equilibrium does not always lead to an efficient assignment of traffic. Let $c_i(q_i) = c_i q_i,\ c_i > 0,\ i = 1,2$ and $B(Q) = 1 - Q$. The corresponding demand functions for the roads are:

$$q_i = \frac{c_{3-i} - (1 + c_{3-i})p_i + p_{3-i}}{(1 + c_1)(1 + c_2) - 1}, \quad i = 1,2.$$

Solving the first order conditions for each firm leads to the Nash equilibrium tolls:

$$p_i = \frac{2c_{3-i}(1 + c_i) + c_i}{4(1 + c_1)(1 + c_2) - 1}, \quad i = 1,2.$$

Replacing in the expression for q_i we can compute the total marginal cost $TMC_i = c_i(q_i) + q_i c_i'(q_i)$. Performing the computations leads to $TMC_i = TMC_j$ if and only if $c_i = c_j$. Hence the traffic assignment in the example is inefficient unless congestion costs are identical across roads. ∎

What is the intuition behind Proposition 4.2? Consider the decision of a road concessionaire. As in any market, the attractiveness of lowering the toll a bit is to attract new users, which yields $p_i \Delta q_i$. The cost, of course, is that users that were already using road i now will pay $q_i \Delta p_i$ less. So it all depends on how many new users will be attracted by a lower toll.

In the standard Bertrand model, lowering the toll an epsilon is enough to attract the whole market. Thus $p_i \Delta q_i$ is large, $q_i \Delta p_i$ is negligible and, as a consequence, price competition is very intense. Congestion, however, implies that road i becomes less attractive at the margin the more users switch to it, thus compensating the lower toll. Roughly speaking, if concessionaire i wants to attract a lot of new users she will have to lower her toll, p_i, a lot as well, to compensate new users for the increased congestion. This softens price competition and, in equilibrium, all concessionaires charge tolls higher than those needed for users to internalize the externality they create.

5 Limit results

In the previous section we have shown that the Nash equilibrium of the game between road owners is inefficient. In this section we show that as the economy becomes large, tolls converge to the socially optimal level. In order to get interesting results, we allow demand for roads to grow at the same rate as capacity expands. Nevertheless, drivers are free to choose any of the roads on any of the networks. Hence, our limit results depends solely on the reduction in the relative size of each *individual* owner with respect to the market and hence on her smaller ability to affect prices.

We begin by considering the replication of complete road networks, where networks are composed of $n \geq 1$ roads with possibly different congestion costs. The first result is that the traffic in each network tends to the efficient assignment as the number of replications increases.[12] Next we consider the special case where networks have only one road and extend the previous result by showing that convergence is monotonous in the number of roads.

Consider the case where there are R identical networks, each one composed of n (possibly asymmetric) roads. The replicated roads are located in parallel, so that a traveller chooses only one road. The following notation is used throughout:

- $q_i^r(R)$: traffic on road i in network r,
- $p_i^r(R)$: the corresponding toll,
- $Q^r(R) \equiv \sum_{i=1}^n q_i^r(R)$: total traffic on network r,
- $Q_R \equiv \sum_{r=1}^R Q^r(R)$: traffic over the R networks.

Demand is assumed to grow at the same rate as capacity. That is, each time that a network is replicated another set of drivers with demand $B(Q)$ is added. Hence the marginal benefit function for the replicated network, denoted by $B_R(Q)$, is related to the marginal benefit function of an individual network by

$$B_R(Q) \equiv B(\frac{Q}{R}).$$

Definition 5.1 *A* symmetric equilibrium *satisfies* $q_i^r(R) \equiv q_i(R)$, $p_i^r(R) \equiv p_i(R)$ *and* $Q^r(R) \equiv Q(R)$ *for all* i, r.

Note that all the results derived in section 4 apply in this section since network replication is a special case of a network with n arbitrary roads. The following proposition shows that when the number of replications is very large, each road in each network approximately carries the optimal number of users and charges the optimal toll.

Proposition 5.1 *Assume the planner's solution, which trivially is independent of the number of replications, is interior. Then for sufficiently large R there exists a unique interior symmetric equilibrium* $(q_i(R), p_i(R), Q(R))$, *and*

$$\lim_{R\to\infty} q_i(R) = q_i^*,$$

$$\lim_{R\to\infty} p_i(R) = q_i^* c'_i(q_i^*).$$

[12] A similar result appears in Scotchmer (1985a) for club goods.

Proof: Whenever it is not a source of confusion, we omit writing out explicit dependence on R. From (2) we have $B_R(Q_R) = p_j^r + c_j(q_j^r)$, and since $B_R(Q_R) = B(Q_R/R) = B(Q)$, it follows that $B(Q) = p_j^r + c_j(q_j^r)$, which, by symmetry, can be written as

$$B(Q) = p_j + c_j(q_j), \quad j = 1, \ldots, n \tag{14}$$

Symmetry also implies that we can write the following partial derivatives independently of the network:

$$\begin{aligned} \frac{\partial q_i^r}{\partial p_i^r} &= \frac{\partial q_i}{\partial p_i} \equiv d_i, \quad i = 1, \ldots, n \\ \frac{\partial q_i^r}{\partial p_j^s} &= \frac{\partial q_i}{\partial p_j} \equiv d_{ij}, \quad s \neq r \text{ or } s = r \text{ and } j \neq i, \;\; i, j = 1, \ldots, n. \end{aligned}$$

Thus d_{ii} denotes the cross partial derivative of traffic with respect to tolls for the same road on different networks, while d_i denotes the own price elasticity for any of the R versions of road i. From Lemma 4.2 it follows that the former is positive while the latter is negative.

¿From (9),

$$d_{ji} = \frac{1 + c'_i(q_i)d_i}{c'_j(q_j)}, \quad j, i = 1, \ldots, n. \tag{15}$$

Applying (4.3) to the composite network consisting of nR roads leads to

$$\frac{1}{R}B'(Q)\left[d_i + R\sum_{j\neq i} d_{ji} + (R-1)d_{ii}\right] = 1 + c_i'(q_i)d_i, \quad i = 1, \ldots, n \tag{16}$$

where we used the fact that $B_R'(Q_R) = B'(Q_R/R)/R = B'(Q)/R$. Now, substituting into (16) the expression for d_{ji} derived in (15) we get:

$$\frac{1}{R}B'(Q)\left[d_i + R\sum_{j\neq i}\frac{1 + c_i'(q_i)d_i}{c_j'(q_j)} + (R-1)\frac{1 + c_i'(q_i)d_i}{c_i'(q_i)}\right] = 1 + c_i'(q_i)d_i, \quad i = 1, \ldots, n;$$

which, solving for d_i leads to:

$$d_i = -\frac{1}{c'_i(q_i)}\left\{1 - \frac{A_i(R)}{R}\right\}, \tag{17}$$

with

$$A_i(R) \equiv \frac{\frac{1}{c'_i(q_i)}}{\sum_{j=1}^n \left[\frac{1}{c'_j(q_j)}\right] - \frac{1}{B'(Q)}} > 0. \tag{18}$$

Since $B' < 0$ and $c'_i > 0$, we have that

$$|A_i(R)| \leq \frac{\frac{1}{c'_i(q_i)}}{\sum_{j=1}^{n} \frac{1}{c'_j(q_j)}} \leq 1,$$

and we can write:

$$d_i = -\frac{1}{c'_i(q_i)} \{1 + O_i(1/R)\} \tag{19}$$

where d_i and c'_i depend on R and $\lim_{R\to\infty} O_i(1/R) = 0,\ i = 1, \ldots, n$.

Now the first order conditions can be written (using symmetry) as:

$$q_i + p_i d_i = 0. \tag{20}$$

Replacing (14) and (19) in the first order conditions and manipulating yields

$$c_i(q_i) + q_i c'_i(q_i) - B(Q) = [B(Q) - c_i(q_i)])O_i(\alpha), \quad i = 1, \ldots, n, \tag{21}$$

where $\alpha \equiv 1/R$. Next we extend the $O_i(\alpha)$ functions to all $\alpha \in [0,1]$ in such a way that the resulting functions are continuously differentiable and then apply the Implicit Function Theorem to the set of equations in (21) at $q_i = q_i^*$ and $\alpha = 0$ (i.e., at the planner's solution). The proof that the corresponding Jacobian is non singular is analogous to that of Lemma 4.1. It follows that (21) has a unique solution for all α in a neighborhood of $\alpha = 0$ (i.e., for all R large enough) and, since the solution is continuous in α, that the corresponding q_i's converge to q_i^* as α tends to zero.

From (14) it follows that:

$$p_i(R) = B\left(\sum q_i(R)\right) - c_i(q_i(R))$$

so that, by continuity of B and c_i,

$$\lim_{R\to\infty} p_i(R) = B\left(\sum q_i^*\right) - c_i(q^*).$$

Comparing the above identity with the planner's first order condition it follows that $p_i(R)$ converges to $q_i^* c'_i(q_i^*)$. ∎

The intuition behind Proposition 5.1 is as follows. As the size of the market increases, the gain in users from a given fall in prices Δp_i increases as well, and becomes very large when the market is large. On the other hand, the cost of lowering p_i, $q_i \Delta p_i$, remains at about the same order of magnitude, because q_i's order of magnitude does not change if both demand and the number of roads are replicated at the same rate. This implies that $p_i \Delta q_i / q_i \Delta p_i$ grows without bound as the market is replicated, thus making price competition tougher and replicating "perfect" competition in the limit.

The previous result shows that the allocation of traffic in a road system where independent road owners set tolls converges to the efficient allocation as the capacity of each road becomes smaller in relation to size of the market. In the special case in which each "network" has a single road we can go further and show that convergence to the efficient allocation is monotonic.

Definition 5.2 *Let $p_1(q) = B(q) - c_1(q)$. We denote the elasticity of demand in an economy with one road by $\eta_1(q) = p_1(q)/[qp_1'(q)]$.*

Proposition 5.2 *Suppose each network consists of a single road, and that $\eta_1^{'}(q) \geq 0$ and $\eta_1(q) \geq -1$ at the equilibrium.*[13] *Then assuming that the planner's solution is interior, in the symmetric equilibrium, as the number of networks R increases, prices and quantities tend monotonically to their efficient values.*

Proof: We drop the subindex from c_1 in what follows. Since in this case (18) becomes

$$A(R) = \frac{B'(q)}{B'(q) - c'(q)},$$

it follows from (17), after some manipulation, that:

$$d_1 = -\frac{1}{c'(q)}\left(1 - R^{-1}\right) + \frac{1}{[B'(q) - c'(q)]}R^{-1}. \tag{22}$$

We now replace (14) and (22) in the first order conditions (20) to obtain:

$$q - [B(q) - c(q)]\left\{\frac{1}{c'(q)}\left[1 - R^{-1}\right] - \frac{1}{[B'(q) - c'(q)]}R^{-1}\right\} = 0$$

Next, since from the definition of η_1 and (2) it follows that

$$\eta_1(q) = \frac{B(q) - c(q)}{q[B'(q) - c'(q)]},$$

the preceding expression leads to:

$$\left(1 + \frac{\eta_1(q)}{R}\right) qc'(q) = \left(1 - \frac{1}{R}\right)[B(q) - c(q)]. \tag{23}$$

Multiplying (23) by R and differentiating across with respect to R, we get:

$$c(q) + qc'(q) - B(q) = -L(q)\frac{dq}{dR}, \tag{24}$$

[13] Engel, Fischer and Galetovic (2001) show that $B^{''} \leq 0$ is sufficient for both conditions on η_1 to hold.

with

(25) $L(q) = qc'(q)\eta_1{}'(q) + [R + \eta_1(q)][c'(q) + qc''(q)] + (R-1)[B'(q) - c'(q)].$

¿From the assumptions on $\eta_1(q)$ and (25) it follows that $L(q) > 0$. Also, the left hand side of (24) is negative by (13). Therefore (24) implies that $\frac{dq}{dR} > 0$. And since (14) implies that

$$\frac{dp}{dR} = [B'(q) - c'(q)]\frac{dq}{dR},$$

we have that $\frac{dp}{dR} < 0$, which completes the proof. ∎

6 Conclusion

Major increases in congestion over the last two decades, combined with a trend toward smaller government, have made private toll roads increasingly attractive in the United States.[14] The standard option for privatizing roads is a Build-Operate-and-Transfer (BOT) contract, where a firm builds and operates the road for a long period of time, and then transfers it back to the government.

When the franchised road has no close substitute, the government must regulate tolls. Yet when there are alternative routes from one place to another, competition between several franchise holders might substitute for regulation. This is the topic we study in this paper, obtaining two main conclusions.

At first sight it would appear that, as in the case of Bertrand competition, toll competition between two roads that are substitutes will lead to tolls set at marginal cost, i.e., zero in our case and excessive congestion. However, lowering tolls raises congestion costs for all users of the road and therefore does not lead to a complete switch of users to the road with the lowest toll. Hence, the owner of each road faces a demand curve that is not infinitely elastic. Thus our first result that competition yields tolls that are higher than optimal and traffic flows that are inefficiently small—there is too little congestion. The result is due to the capacity constraint in roads, i.e., it is related to the idea that a price game when there are capacity constraints does not lead to the Bertrand result but is closer to a Cournot equilibrium (see Kreps and Scheinkman [1983]).

It is interesting to note that a similar result holds for a toll road that is a substitute of a public untolled road. The owner of the tolled road will be able to exact a positive toll, given sufficient congestion on the alternative road. A decrease in congestion in the untolled road hurts the private road. Hence, its owner will oppose all attempts to increase the capacity of the untolled road. For example, in the

[14]"Viewing private participation as a source of badly needed capital, officials in some states have welcome private investment in toll roads." Congressional Budget Office (1998, p. xii).

case of the Dulles Greenway near Washington D.C., a toll road that joins Dulles Airport to Leesburg in Virginia, the owner of the road has opposed the expansion of competitive public freeways which are untolled.[15]

The second result shows that as the number of independently owned roads increases the increased number of participants in the market makes road system more competitive, even if demand increases at the same rate. In fact, in the limit both total traffic flow and traffic allocation will be efficient. This suggests that in some cases where there is more than one road joining two cities or parts of a city, toll competition may be a viable way of regulating private roads.

Last, there is close relationship between competition between private toll roads and competition between clubs, which may allow the transfer of results between these two fields.

[15] See Viton (1995), Verhoef et al. (1996) and de Palma and Lindsay (2000) for analyses of a private road competing with an untolled alternative.

APPENDIX

A Proof of Proposition 3.1

Let $1_{n,n}$ and $D(\lambda_1, \ldots, \lambda_n)$ denote, respectively, a n by n matrix with all elements equal to 1 and a diagonal matrix with i-th element on the diagonal equal to λ_i. A straightforward calculation shows that the Hessian of S may be written as

$$H = k1_{n,n} - D(\lambda_1, \ldots, \lambda_n) \tag{26}$$

where $k \equiv B'(\sum q_i)$ and $\lambda_j \equiv 2c_j'(q_j) + q_j c_j''(q_j)$.

Given a column vector $x' = (x_1, \ldots, x_n)$ we have that

$$x'Hx = k(\sum_i x_i)^2 - \sum_i \lambda_i x_i^2.$$

Since $k < 0$ and all $\lambda_j > 0$, it suffices that one of the x_i's differ from zero to have $x'Hx < 0$. Thus $\mathcal{S}$ is strictly concave.

Since $B' < 0$, we have that

$$\frac{\partial \mathcal{S}}{\partial q_i} = B(\sum_{j=1}^{n} q_j) - c_i(q_i) - q_i c_i'(q_i) \leq B(q_i) - c_i(q_i) - q_i c_i'(q_i). \tag{27}$$

It then follows from assumption (4) that there exist $\overline{q}_1, \ldots, \overline{q}_n$ such that the partial derivative evaluated at $q = (q_1, \ldots, q_n)$ is negative if any of the q_i's is larger than the corresponding $\overline{q}_i$. Hence we may restrict maximization of $\mathcal{S}$ to a compact subset of the positive orthant. Since a continuous function over a compact set has a maximum, it follows that $\mathcal{S}$ has a maximum. Due to strict concavity of $\mathcal{S}$ this maximum, denoted by q^*, is unique. ∎

B Proof of Lemma 4.1

A straightforward calculation shows that the Jacobian matrix corresponding to p as a function of q is of the form

$$J \equiv k1_{n,n} - D(\mu_1, \ldots, \mu_n),$$

where the notation is the same as in the proof of Proposition 3.1 and $\mu_i = c_i'(q_i) > 0$, $i = 1, \ldots, n$.

Denote the i-th column of the Jacobian matrix by J_i. To show that the Jacobian is non singular, we show that if $\alpha_1, \ldots, \alpha_n$ are real numbers such that $\sum_i \alpha_i J_i = 0_n$, where 0_n denotes the vector in R^n with all coordinates equal to zero, then all the α_i's are equal to zero.

A straightforward calculation shows that $\sum_i \alpha_i J_i = 0_n$ implies that:

$$k(\sum_i \alpha_i) = \alpha_1 \mu_1 = \ldots = \alpha_n \mu_n. \tag{28}$$

Hence:

$$k(\sum_i \alpha_i)\frac{1}{\mu_j} = \alpha_j, \quad j = 1, \ldots, n.$$

Summing over j leads to:

$$k\left(\sum_i \alpha_i\right) \sum_j \frac{1}{\mu_j} = \sum_j \alpha_j. \tag{29}$$

Since all μ_j are positive, it follows from (28) that all α_i have the same sign. If all α_i are strictly positive or all α_i are strictly negative, it follows from (29) that

$$k \sum_j \frac{1}{\mu_j} = 1$$

which cannot hold since $k < 0$ and the $\mu_j > 0$. It follows that all $\alpha_i = 0$.

Having shown that the Jacobian is non singular, we may now apply the Implicit Function Theorem to conclude that the inverse function is well defined and differentiable. ∎

C Proof of Lemma 4.2

From (2), $p_j + c_j(q_j) = p_i + c_i(q_i),\ j \neq i$. Differentiating with respect to p_i leads to the identity in (8). Differentiating both sides of (2) with respect to p_i leads to the identity in (9).

The two identities we just proved imply that:

$$c'_j(q_j)\frac{\partial q_j}{\partial p_i} = B'(Q)\sum_{k=1}^{n}\frac{\partial q_k}{\partial p_i}, \quad j \neq i. \tag{30}$$

Next we prove (i) and (ii). From the identity in (9) and the assumption that all $c'_j > 0$ we have that all $\partial q_j/\partial p_i$ have the same sign, $j \neq i$. If this common sign were negative, (9) implies that $\partial q_i/\partial p_i < 0$ and the left hand side of (30) would be negative while the corresponding right hand side was positive. If all $\partial q_j/\partial p_i$ were equal to zero, $j \neq i$, then (30) and the assumption that $B' < 0$ imply that $\partial q_i/\partial p_i = 0$. Yet then the right hand side of (9) would be positive while the left hand side is zero. We conclude that $\partial q_j/\partial p_i > 0$ for all $j \neq i$. It then follows from (30) that $\partial q_i/\partial p_i < 0$, for otherwise the left hand side of (30) would be positive while the right hand side was negative. We have thus shown (i) and (ii). The inequalities in (9) and (4.3) now follow trivially. ∎

D Proof of Proposition 4.1

To the notation introduced when stating the proposition add:

$$\begin{aligned}
S(k) &\equiv \sum_{i=1}^{n} k_i, \\
\tilde{B} &\equiv \frac{1}{B'(\sum_{k=1}^{n} q_k)}, \\
A_i &\equiv \frac{k_i}{S(k) - \tilde{B}}, \\
\gamma_i &\equiv \frac{c_i''(q_i)}{[c_i'(q_i)]^3}, \\
S(\gamma) &\equiv \sum_{i=1}^{n} \gamma_i, \\
\beta^* &\equiv -\frac{B''(\sum_{l=1}^{n} q_l)}{[B'(\sum_{l=1}^{n} q_l)]^2}.
\end{aligned}$$

Some patient, but straightforward calculations then show that:

$$\frac{\partial q_i}{\partial p_i} = -k_i(1-A_i), \quad i=1,\dots,n, \tag{31}$$

$$\frac{\partial q_l}{\partial p_i} = k_l A_i, \quad l \neq i, i=1,\dots,n, \tag{32}$$

$$\frac{\partial k_j}{\partial p_i} = -\gamma_j A_i, \quad j \neq i, i=1,\dots,n; \tag{33}$$

$$\frac{\partial k_i}{\partial p_i} = \gamma_i(1-A_i), \quad i=1,\dots,n; \tag{34}$$

$$\sum_{l=1}^{n}\frac{\partial k_l}{\partial p_i} = \gamma_i - S(\gamma)A_i, \tag{35}$$

$$k_i - S(k)A_i = \frac{-k_i\tilde{B}}{\sum_{l=1}^{n} k_l - \tilde{B}} > 0, \tag{36}$$

$$\sum_{l=1}^{n}\frac{\partial q_l}{\partial p_i} = -[k_i - S(k)A_i] < 0, \tag{37}$$

$$\frac{\partial \tilde{B}}{\partial p_i} = -\beta^*[k_i - S(k)A_i], \tag{38}$$

$$\frac{\partial A_i}{\partial p_i} = \frac{A_i}{k_i}\left\{\gamma_i(1-A_i) - A_i[\gamma_i - S(\gamma)A_i] - \beta^* A_i[k_i - S(k)A_i]\right\}. \tag{39}$$

Next we prove strict concavity of the profit function of the i-th road's owner. This implies the existence of a Nash equilibrium in pure strategies (see Mas-Colell, Whinston and Green, 1995, p. 260).

Denoting profits of firm i by $\Pi_i(p_i)$ we have that:

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = 2\frac{\partial q_i}{\partial p_i} + p_i\frac{\partial^2 q_i}{\partial p_i^2}.$$

Since $\partial q_i/\partial p_i < 0$, a sufficient condition for strict concavity of the profit function (and therefore existence of a Nash equilibrium) is that $\partial^2 q_i/\partial p_i^2 \leq 0$.

A calculation based on the expressions derived above shows that:

$$\frac{\partial^2 q_i}{\partial p_i^2} = -\gamma_i(1-A_i)^3 + [S(\gamma)-\gamma_i]A_i^3 - \beta^*[k_i - S(k)A_i]A_i^2. \tag{40}$$

From (31) it follows that $k_i - S(k)A_i > 0$. Some patient algebra shows that

$$\gamma_i(1-A_i)^3 - [S(\gamma)-\gamma_i]A_i^3 \geq 0$$

if and only if

$$\frac{\gamma_i}{S_{-i}(\gamma)} \geq \left[\frac{k_i}{S_{-i}(k) + |\tilde{B}|}\right]^3, \tag{41}$$

where $S_{-i}(k) \equiv \sum_{j\neq i} k_j$, $S_{-i}(\gamma)$ analogous.

And since $\gamma_i = k_i^3 c_i''$, where c_i'' is evaluated at q_i, condition (41) is equivalent to (10). It now follows from (40) that a sufficient condition for strict concavity of the profit function is that $B'' \leq 0$ (so that $\beta^* \geq 0$) and (40), thereby concluding the proof. ∎

Next some particular cases where the assumptions of Proposition 4.1 hold are presented. In the linear case $\partial^2 q_i/\partial p_i^2 = 0$, so that Π_i is strictly concave without a further do. The case of quadratic time-cost is more interesting. Assume

$$c_i(q_i) = \alpha_0^{(i)} + \alpha_1^{(i)} q_i + \frac{1}{2}\alpha_2^{(i)} q_i^2,$$

with $\alpha_0^{(i)} > 0$, $\alpha_1^{(i)} > 0$ and $\alpha_2^{(i)} > 0$, for all i.[16]

Define η via:

$$\eta \equiv \frac{\max_i \alpha_2^{(i)}}{\min_i \alpha_2^{(i)}}.$$

Then a sufficient condition for (10) to hold is:

$$|\tilde{B}|^3 \geq (\eta - 1)\sum_{j\neq i} k_j^3, \qquad \forall i \tag{42}$$

Two particular cases are of interest. First, when all $\alpha_2^{(i)}$'s are identical. In this case $\eta = 1$ and existence follows for any function $B(Q)$ that is concave and decreasing. Second, if $B(Q) = B_0 - B_1 Q$, with $B_0 > 0$, $B_1 > 0$, then Proposition 4.1 implies that

$$\frac{1}{B_1^3} \geq (\eta - 1)\sum_{j\neq i} \frac{1}{[\alpha_1^{(j)}]^3}, \qquad \forall i$$

is sufficient for existence of a Nash equilibrium.

[16]The non-trivial assumption is $\alpha_1^{(i)} > 0$, we want $c_i'(0) > 0$.

References

Arnott, R., A. de Palma and R. Lindsey, "Economics of a Bottleneck," *Journal of Urban Economics* **27**, 111–130, 1990

Arnott, R., A. de Palma and R. Lindsey, "A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand," *American Economic Review* **83**, 161–179, 1993

Arnott, R., A. de Palma and R. Lindsey, "The Welfare Effects of Congestion Tolls with Heterogeneous Commuters," *Journal of Transport Economics and Policy* **28**, 139–161, 1994

Berglas, E., "On the Theory of Clubs," *American Economic Review Papers and Proceedings*, 116–121, 1976

Berglas, E., "The Market Provision of Club Goods Once Again," *Journal of Public Economics* **15** 389–393, 1981

Berglas, E., and D. Pines, "Clubs, Local Public Goods and Transportation Models: A Synthesis," *Journal of Public Economics* **15**, 141–162, 1981

Boadway, R., "A Note on the Market Provision of Club Goods," *Journal of Public Economics* **13**, 131–137, 1980

Congressional Budget Office Study, "Innovative Financing of Highways: An Analysis of Proposals," Washington, DC: Congress of the United States, January 1998.

Engel, E., R. Fischer and A. Galetovic, "Least-Present-Value-of-Revenue Auctions and Highway Franchising," *Journal of Political Economy*, **109** (5), 993–1020, 2001

Häckner, J. and Nyberg, S., "Vanity and Congestion: A Study of Reciprocal Externalities", *Economica* **63**, 97-111, 1996.

Hau, T., "Economic Fundamentals of Road Pricing: A Diagrammatic Analysis," PR Working Paper 1070, Washington: The World Bank, 1992

Kreps, D. and J. Scheinkman, "Quantity Precommitments and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics* **14**, 326–337, 1983

Levhari, D. and I. Luski., "Duopoly Pricing and Waiting Lines," *European Economic Review* **11**, 17-35 (1978).

Lévy-Lambert, H., "Tarification des Services a Qualité Variable: Application aux Péages de Circulation," *Econometrica* **36**, 564–574, 1968

Marchand, M. "A Note on Optimal Tolls in an Imperfect Environment," *Econometrica* **36**, 575–581, 1968

Mas-Colell, A., M. Whinston and J.R. Green, *Microeconomic Theory*, New York: Oxford University Press, 1995.

Mohring, H. (ed.), *The Economics of Transport*. Aldershot: Elgar, 1994

de Palma, A., "A Game-Theoretic Approach to the Analysis of Simple Congested Networks," *American Economic Review Papers and Proceedings*, 494–500, 1992

de Palma, A. and R. Lindsay, "Private Toll Roads: Competition Under Various Ownership Regimes," *Annals of Regional Science* **34**, 13-35, 2000

Reitman, D., "Endogenous Quality Differentiation in Congested Markets", *The Journal of Industrial Economics* **39**, 621-647, 1991

Scotchmer, S., "Profit Maximizing Clubs," *Journal of Public Economics* **27**, 25–45, 1985a

Scotchmer, S., "Two-Tier Pricing of Shared Facilities in a Free Entry Equilibrium," *Rand Journal of Economics* **16**, 456–472, 1985b

Sheffi, Y., *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*, Englewood Cliffs: Prentice-Hall, 1985.

Verhoef, E., P. Nijkamp and P. Rietveld, "Second-Best Congestion Pricing: The Case of an Untolled Alternative" *Journal of Urban Economics* **40**, 279–302, 1996

Vickrey, W., "Congestion Theory and Transport Investment," *American Economic Review Papers and Proceedings* **59**, 251–261, 1969

Viton, P., "Private Roads," *Journal of Urban Economics* **37**, 260–289, 1995

Walters, A. "The Theory and Measurement of Private and Social Cost of Highway Congestion," *Econometrica* **29**, 676–699, 1961

Wardrop, J., "Some Theoretical Aspects of Road Traffic Research," *Proceedings of the Institute of Civil Engineers* **1**, 325–378, 1952